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Localized Spin Fluctuations in Nearly-Magnetic Dilute Alloys: Thermal Resistivity and Lorenz Number*

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We calculate the thermal resistivity and Lorenz number due to scattering of conduction electrons from localized spin fluctuations, using a model appropriate for nearly-magnetic transition-metal impurities in a host of similar electronic structure. The temperature dependence of the Lorenz number, which is determined by the characteristic energy of the spin-fluctuation excitation spectrum, is predicted for *Ir*Fe and related alloys assuming the spin-fluctuation model is applicable.

I. INTRODUCTION

We have shown[1] that a good account of the anomalous electrical resistivity of *Ir*Fe and related alloys is given by the scattering of conduction electrons from localized spin fluctuations (local paramagnons). In this paper we make an analogous calculation of the thermal resistivity and Lorenz number.

The spin-fluctuation model we use was first proposed by Lederer and Mills[2] to explain the T^2 term in the electrical resistivity of dilute *Pd*Ni alloys. It applies to dilute transition-metal alloys with nearly magnetic impurities in which the host and impurity have similar electronic structure, the interference between potential scattering and spin-fluctuation scattering being neglected. The intra-atomic Coulomb interaction U_0 between opposite-spin d electrons in the host is considered to be increased to a value $(U_0+\delta U)$ in the impurity cell, the Hamiltonian being

$$H=T+U_0\sum_i n_{i\uparrow}n_{i\downarrow}+\delta U\sum_j n_{j\uparrow}n_{j\downarrow}\ .$$

Here T is the kinetic energy and $n_{i\sigma}=c_{i\sigma}^\dagger c_{i\sigma}$ is the number operator for d electrons of spin σ, $c_{i\sigma}^\dagger$ and $c_{i\sigma}$ being the creation and destruction operators for the ith-cell Wannier orbital. The sum i is taken over all cells, while sum j is over just the impurity cells. Lederer and Mills calculated the generalized susceptibility response function for this Hamiltonian in the random-phase approximation. The spectral density for the spin fluctuations is given by the imaginary part of this response function.

The model should be appropriate for nearly-magnetic $3d$ impurities in $4d$ and $5d$ hosts from the same or nearby columns. For instance, the behavior of Fe impurities in the $4d$ series Ru-Rh-Pd changes from nonmagnetic in Ru to ferromagnetic in Pd as the exchange enhancement in the host increases. For the intermediate case of Fe impurities in Rh (and in the corresponding $5d$-metal Ir), we suggested that the temperature dependence of the resistivity and susceptibility could be understood in terms of strong localized spin fluctuations at the Fe sites.

Although the model is too oversimplified to allow quantitative predictions from first principles, it does seem capable of accounting for the main qualitative features of the experimental data. We calculated[1] a universal curve for the electrical resistivity showing a gradual change from a T^2 to a linear dependence at temperatures around $0.25T_s$, where T_s is the characteristic temperature of the localized spin fluctuations in the alloy. Fitting this universal curve gave values of T_s for *Ir*Fe ($T_s\sim 30\,^\circ$K), *Rh*Fe ($T_s<1.5\,^\circ$K), and *Pd*Ni ($T_s\sim 80\,^\circ$K). The essential feature of the model applied to *Ir*Fe and especially *Rh*Fe is the small energy deduced for the localized spin fluctuations.

This paper suggests a method of testing the localized spin-fluctuation model proposed for *Ir*Fe and *Rh*Fe by measuring the thermal resistivity and Lorenz number. At low temperatures, the so-called "vertical processes" due to inelastic scattering by spin fluctuations make a large contribution to the thermal resistivity, and so reduce the Lorenz number below the classical value, as shown by Schriempf, Schindler, and Mills (SSM).[3] However, the effect of these vertical processes is greatly reduced as the temperature increases above T_s, since the spin fluctuations then have insufficient energy to scatter electrons through the thermal layer. Hence, the increase of the Lorenz number with temperature indicates directly the order of magnitude of the energy of the excitations producing the inelastic scattering, i.e., gives a check on the order of magnitude of T_s if the excitations are assumed to be spin fluctuations.

It has been suggested[4] that the electrical resistivity due to s-d exchange scattering from magnetic impurities may decrease as temperature decreases in the presence of strong nonmagnetic scattering (in contrast to the more usual opposite behavior[5]), and that *Rh*Fe and *Ir*Fe are Kondo systems of this type (although potential scattering is weak in these alloys). If this were the case, fitting the measured susceptibilities of these alloys[6] to a Curie-Weiss law indicates that the Kondo temperatures[5] (about 10 °K for *Rh*Fe and 100 °K for *Ir*Fe) are considerably larger than the corresponding values of T_s if the localized spin-fluctuation model is applicable. The temperature dependence of the Lorenz number might therefore provide evidence to determine whether the Kondo model or the localized spin-fluctuation model is more appropriate for *Ir*Fe and *Rh*Fe.

SSM[3] have made accurate measurements of the thermal resistivity of dilute *Pd*Ni alloys below 20 °K and have shown that the Lorenz number for electron-electron scattering is small, as predicted by their calculation, which is valid for temperatures $T \ll T_s$. We extend the calculation to higher temperature ranges to obtain predictions for *Ir*Fe and *Rh*Fe. Since the presence of lattice conductivity reduces the accuracy of measurements of electronic thermal resistivity, we consider the effect of lattice conductivity on the separation of thermal resistivity components, with reference to the *Pd*Ni data of SSM.[3]

We mention that a linear T term in the thermal resistivity (corresponding to a T^2 term in the electrical resistivity) has been observed at low temperatures in several transition metals[7] besides *Pd*Ni alloys: It is found that the Lorenz number for electron-electron scattering is reduced below the classical Sommerfeld value. To account for this behavior, several calculations have been made of the thermal resistivity and Lorenz number for electron-electron scattering,[8] and in particular for scattering from uniform band paramagnons at low temperatures[9,10]; the relation of these calculations to the present localized spin-fluctuation model has been discussed in some detail by SSM.[3] The meaning of "vertical" and "horizontal" processes for inelastic scattering from spin fluctuations has also been fully discussed by SSM.[3]

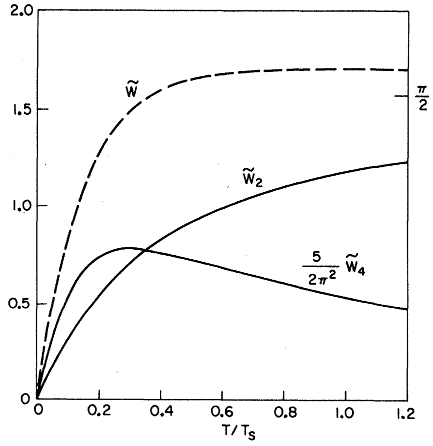

FIG. 1. Universal curves for the thermal resistivity due to scattering by localized spin fluctuations, for the horizontal ($\tilde{W}_2$) and vertical ($\tilde{W}_4$) components of scattering. The total localized spin-fluctuation resistivity $\tilde{W}$ is shown for the case $a_1/a_3 = 1$.

II. LORENZ NUMBER

The calculation of the thermal resistivity W_m due to scattering of conduction electrons from d-electron spin-density fluctuations localized at impurity sites is given in the Appendix. We find that the normalized thermal resistivity $\tilde{W}$ for spin-fluctuation scattering is

$$\tilde{W} = \tilde{W}_2 + \frac{3}{\pi^2}\left(\frac{a_1}{a_3} - \frac{1}{6}\right)\tilde{W}_4 . \qquad (1)$$

The integrals $\tilde{W}_2$ and $\tilde{W}_4$ are defined by (A20) and plotted in Fig. 1 as a function of normalized temperature $\tilde{T} = T/T_s$, where $k_B T_s$ is the energy of the peak in the spin-fluctuation excitation spectrum. Thus, for thermal resistivity there are two universal curves, one arising from "horizontal" processes ($\tilde{W}_2$) and the other from "vertical" processes ($\tilde{W}_4$).

In a vertical process, which contributes to the thermal but not the electrical resistivity, the energy of a conduction electron is changed by inelastic scattering from a spin fluctuation. For instance, the heat current carried by a "hot" electron is reversed if it is scattered through the Fermi surface to become a "cold" electron, although the electrical current is not significantly changed unless the direction of motion of the electron is changed, i.e., unless there is significant horizontal scattering. The spin-fluctuation thermal resistivity is the sum of the vertical and horizontal components, whose relative magnitude is rather difficult to predict since it is determined by the value of the ratio a_1/a_3 (defined in the Appendix) which depends on the angular distribution of scattering.

Since only the horizontal component of scattering contributes to electrical resistivity, we see that

$$\tilde{\rho} = \tilde{W}_2 \tilde{T} , \qquad (2)$$

where $\tilde{\rho}$ is the universal curve for the electrical resistivity[1] due to localized spin fluctuations. Taking the ratio of electrical resistivity ρ_m and thermal resistivity W_m due to scattering by spin fluctuations, we obtain the Lorenz number L_m for localized spin-fluctuation scattering

$$L_m = \frac{\rho_m}{W_m T} = L_0 \frac{\tilde{\rho}}{\tilde{W}\tilde{T}} = L_0 \left[1 + \frac{3}{\pi^3}\left(\frac{a_1}{a_3} - \frac{1}{6}\right)\frac{\tilde{W}_4}{\tilde{W}_2}\right]^{-1} , \qquad (3)$$

where L_0 is the classical Sommerfeld value of the Lorenz number:

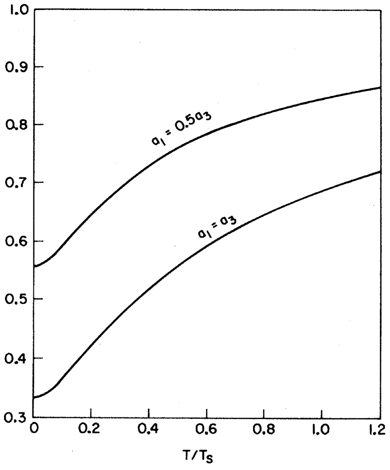

FIG. 2. Lorenz number L_m for scattering due to localized spin fluctuations.

$$L_0 = \pi^2 k_B^2 / 3e^2 . \tag{4}$$

Here k_B is Boltzmann's constant and e the electronic charge. At low temperatures

$$L_m(T \to 0) = L_0 5(3 + 12a_1/a_3)^{-1} , \tag{5}$$

which is the temperature-independent Lorenz number calculated by SSM.[3]

Two examples of L_m for different values of the ratio a_1/a_3 are shown in Fig. 2. From (2), ρ_m and the horizontal scattering term of W_m are related at all temperatures by the classical Lorenz number L_0, so the deviation of L_m from the value L_0 arises from the presence of the vertical scattering component. Hence $L_m \to L_0$ at high temperatures where the contribution of the vertical component $\tilde{W}_4$ tends to zero. The horizontal resistivity component has its largest magnitude for the case of uniform angular scattering, in which case $a_1/a_3 = 0.5$, according to (A10) below. Hence, the curve for $a_1/a_3 = 0.5$ in Fig. 2 represents the upper bound of L_m in our model.

The variation with temperature of the reduction of L_m below L_0 indicates the energy of the excitations involved in the inelastic scattering processes which give rise to the vertical component of thermal resistivity. Comparison of experimental measurements of L_m with Fig. 2 should indicate the energy $k_B T_s$ of the spin-fluctuation excitation spectrum.

III. SEPARATION OF RESISTIVITY COMPONENTS

Making the usual assumption that the different components of electronic thermal conductivity are additive, and neglecting the lattice conductivity, we obtain the apparent resistivity due to localized spin fluctuations as

$$W_m' = W_{\text{imp}} - C_{\text{imp}}/T = W_{\text{alloy}} - W_{\text{host}} - C_{\text{imp}}/T , \tag{6}$$

where W_{alloy} and W_{host} are the measured thermal resistivities in the alloy and in the host, and C_{imp}/T is the thermal resistivity due to nonmagnetic scattering by impurities. As for the electrical resistivity, it is assumed that the electron-phonon resistivity component is the same in the host metal and in the alloys.

Figure 3 shows the data of SSM[3] for *Pd*Ni alloys replotted to illustrate the method of analysis suggested by (6). We plot $W_m' T$ rather than W_m' so that the nonmagnetic component is a constant C_{imp}, which can easily be subtracted off. The data are consistent with this method of analysis, since W_m' is approximately linear in impurity concentration c, and in temperature below 15 °K (as expected for *Pd*Ni since the electrical resistivity data[9,1] suggest $T_s \sim 80$ °K).

The presence of lattice conductivity will reduce W_m'. At low temperatures, the lattice conductivity is expected[11] to vary as T^2:

$$(W_{\text{lat}})^{-1} = DT^2 , \tag{7}$$

where D is a constant. Fletcher and Greig[12] have deduced values of $(W_{\text{lat}})^{-1}$ in *Pd*Ag alloys with Ag concentrations from 5% to 20%. These measurements suggest[13] that D for dilute Pd alloys is of the order of 3×10^{-5} to 10^{-4} (W/cm)/°K^3. The lattice conductivity in dilute *Pt*Au and *Pt*Ir alloys is similar.[12]

The total thermal resistivity W in an alloy or host with electronic resistivity W_{el} and lattice conductivity (7) is

$$W = W_{\text{el}}(1 + W_{\text{el}} DT^2)^{-1} . \tag{8}$$

Figure 4 shows the effect of lattice conductivity reducing the apparent resistivity W_m' deduced from (6) below the actual spin-fluctuation resistivity W_m (taken as linear in T) for parameter values corresponding approximately to *Pd*Ni, with $D = 10^{-4}$

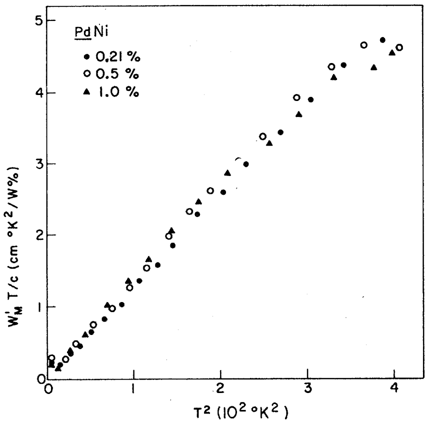

FIG. 3. Thermal resistivity component W_m' for *Pd*Ni alloys derived from the data of SSM[3] using (6).

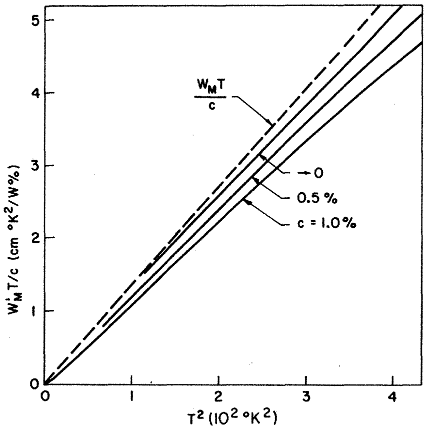

FIG. 4. Apparent thermal resistivity W'_m for localized spin-fluctuation scattering, showing the effect for different impurity concentrations c of a lattice conductivity (7) with $D = 10^{-4}$(W/cm)/°K³. As an approximation for *Pd*Ni below 20 °K, W_{el} is taken as $W_{el} = (1.1 + 13.8c)/T + (3.0 + 13.5c) \times 10^{-3}\, T + 2.55 \times 10^{-4}\, T^2$ cm °K/W, where T is measured in °K and c in atomic percent.

(W/cm)/°K³ taken as the same in pure Pd and the alloys.

The effect of lattice conductivity is roughly proportional to D and to impurity concentration c. Assuming that the addition of impurities affects the electronic conductivity much more strongly than the lattice conductivity, the magnitude of the reduction of W'_m/c by the lattice conductivity will increase as c increases (although taking $C_{\rm imp}$ as the intercept on a plot of $W'_m T$ vs T^2 will tend to reduce this c dependence).

Since the *Pd*Ni data in Fig. 3 show little concentration dependence, it is unlikely that the reduction in slope of $W'_m T/c$ vs T^2 due to lattice conductivity is more than 10%. Taking $W_m/c = 1.32 \times 10^{-2}\, T$ (cm/W)/% for $T < 15$ °K, and by an analogous analysis of electrical resistivity data[3] $\rho_m/c = 1.45 \times 10^{-4}\, T^2$ ($\mu\Omega$ cm/°K²)/%, we get for *Pd*Ni when $T \ll T_s$:

$$L_m = 1.1\begin{pmatrix} +0.1 \\ -0.2 \end{pmatrix} \times 10^{-8}\ \mathrm{V}^2/{}^\circ\mathrm{K}^2\ , \qquad (9)$$

so that L_m/L_0 is between 0.35 and 0.50. To observe the increase in L_m predicted by our calculation, it would be necessary to extend the *Pd*Ni measurements to higher temperatures (of the order of T_s).

The value of L_m in (9) is similar to the value of the Lorenz number for all electron-electron scattering (i.e., including scattering by spin fluctuations in the host Pd) obtained by SSM,[3] using a rather different method of analysis, for each of the *Pd*Ni alloys investigated. In contrast, the Lorenz numbers for nonmagnetic scattering from impurities were found[3] to be within 2% of the classical value L_0.

IV. DISCUSSION

We have extended the calculation by SSM[3] of the Lorenz number L_m for spin-fluctuation scattering to temperatures of order T_s and larger. The curves in Fig. 2 illustrate how the temperature dependence of L_m depends on T_s and on the magnitude of the reduction in L_m. This reduction of L_m below L_0 is caused by the contribution of vertical processes to the thermal resistivity and its magnitude depends on the angular distribution of scattering, which cannot be reliably predicted by our isotropic model. However, if the magnitude of L_m as $T \to 0$ is determined experimentally, the increase of L_m as T increases indicates the value of T_s for localized spin fluctuations in the alloy.

For *Ir*Fe the electrical resistivity data[14] indicate that $T_s \sim 30$ °K, so this alloy should be suitable for checking the predicted increase of L_m with T. For *Rh*Fe the T^2 dependence of electrical resistivity was not observed,[15] because the value of T_s is apparently much smaller (below 1.5 °K), so the linear T dependence of W_m may not be observable. We can make the interesting prediction that L_m should be approximately equal to L_0 down to very low temperatures for *Rh*Fe if the spin-fluctuation model is applicable.

Fortunately, the lattice conductivity in the transition metals Pd and Pt is roughly an order of magnitude smaller than in the noble metals.[12] It should be possible to obtain useful data for L_m in *Ir*Fe and *Rh*Fe if the lattice conductivity in these alloys is no larger than in the *Pd*Ni alloys[3] discussed in Sec. III. To identify the effects of lattice conductivity on W'_m, it is desirable to make measurements at several different impurity concentrations. Another possible complication is deviations from additivity of the different thermal resistivity components,[16] although the constancy of the Lorenz number for the *Pd*Ni data of SSM[3] suggests that, at least for this case, such deviations are not too significant.

We conclude that measuring the Lorenz number L_m for *Ir*Fe and *Rh*Fe should show whether the excitations producing the resistivities ρ_m and W_m are consistent with the low-energy localized spin fluctuations postulated[1] to explain the temperature dependence of ρ_m.

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APPENDIX

In this Appendix we use the standard variational procedure[3,17] to calculate the thermal resistivity

W_m due to scattering of conduction electrons from localized spin fluctuations. To represent this scattering process we employ a simple isotropic two-band model, one band (s-like) contributing to the conductivity and the other (d-like) giving rise to magnetic properties. Then the thermal resistivity W_m is

$$W_m = \frac{27\hbar^2}{8\pi^4 k_F^4 k_B (k_B T)^4} \iint dk\, dk'\, f(\epsilon_k)[1 - f(\epsilon_{k'})] \times \int_0^{2k_F} q\, dq\, Q(\vec{\mathrm{k}} \rightarrow \vec{\mathrm{k}}{}')(\omega^2 k_F^2 + \epsilon_k \epsilon_{k'} q^2) \quad , \tag{A1}$$

where $Q(\vec{\mathrm{k}} \rightarrow \vec{\mathrm{k}}{}')$ is the scattering rate from filled conduction-electron state ϵ_k to empty state $\epsilon_{k'}$, $f(\epsilon_k)$ is the Fermi-Dirac function, k_F is the Fermi wave vector for the conduction band, q is the magnitude of the change $\vec{\mathrm{q}}$ in the wave vector upon scattering:

$$\vec{\mathrm{q}} = \vec{\mathrm{k}}{}' - \vec{\mathrm{k}} \ , \tag{A2}$$

and ω is the change in energy:

$$\omega = \epsilon_{k'} - \epsilon_k \ . \tag{A3}$$

As for electrical resistivity,[1] (A1) is an average over initial and final states $\vec{\mathrm{k}}$ and $\vec{\mathrm{k}}{}'$ of the scattering rate $Q(\vec{\mathrm{k}} \rightarrow \vec{\mathrm{k}}{}')$, but because of the contribution of vertical processes depending on energy change ω, the weighting factor for $Q(\vec{\mathrm{k}} \rightarrow \vec{\mathrm{k}}{}')$ is not simply the change in forward momentum (which is proportional to q^2).

Expressing $Q(\vec{\mathrm{k}} \rightarrow \vec{\mathrm{k}}{}')$ in terms of the spectral density $A(q, \omega)$ for localized spin fluctuations,[1,2] we obtain

$$W_m = \frac{\rho_0}{L_0} \frac{3k_B\beta^4}{\pi^2} \int_0^\infty d\omega\, n(\omega)[\omega^2 I_0(\omega)\bar{A}_1(\omega) + \omega I_1(\omega)\bar{A}_3(\omega) + I_2(\omega)\bar{A}_3(\omega)] \ , \tag{A4}$$

where $\beta = (k_B T)^{-1}$, and $n(\omega)$ is the Bose function. As before,[1] ρ_0 is

$$\rho_0 = \left(\frac{JN(\epsilon_F)}{4}\right)^2 \frac{\nu}{n} \frac{m}{ne^2\tau_F} \ , \tag{A5}$$

where J is the coupling constant for scattering of conduction electrons from localized spin fluctuations, ν and n are the number of atoms and conduction electrons, respectively, per unit volume, $N(\epsilon_F)$ is the density of conduction electrons at the Fermi energy ϵ_F, m is their effective mass, and $\tau_F = \hbar/\epsilon_F$. The $\bar{A}_n(\omega)$ is the weighted average over wave vector of the spectral density:

$$\bar{A}_n(\omega) = (1/k_F^{n+1}) \int_0^{2k_F} dq\, q^n |F(q)|^2 A(q, \omega) \ , \tag{A6}$$

where $F(q)$ is a form factor for the impurity d orbital, and

$$I_n(\omega) = \int_{-\infty}^{+\infty} d\epsilon_k\, f(\epsilon_k)[1 - f(\epsilon_k + \omega)]\epsilon_k^n \ . \tag{A7}$$

$I_n(\omega)$ is easily evaluated for $k_B T \ll \epsilon_F$, giving

$$W_m = \frac{\rho_0 k_B \beta^4}{L_0} \int_0^\infty d\omega \times \frac{\beta^{-2}\omega\bar{A}_3(\omega) + 3\pi^{-2}\omega^3\bar{A}_1(\omega) - (2\pi^2)^{-1}\omega^3\bar{A}_3(\omega)}{(e^{\beta\omega} - 1)(1 - e^{-\beta\omega})} \ . \tag{A8}$$

We can give a physical explanation of the three terms in this expression. The first term, which includes in $\bar{A}_3(\omega)$ the weighting factor q^2 for angular scattering, represents the horizontal component of scattering, and so is the same as in the expression for electrical resistivity. The second term represents the vertical scattering component, having an additional energy-weighting factor of ω^2 but not including the q^2 factor in $\bar{A}_1(\omega)$. The third term is a negative cross term having both the angular- and energy-weighting factors q^2 and ω^2, which arises as follows (cf. Ziman,[17] p. 390). In a scattering process in which the vertical and horizontal components are both large, the electron is changed from, say, a hot electron moving in one direction to a cold electron moving in the opposite direction, so there is little net effect on the heat current. Hence, the total resistivity is less than the sum of the vertical and horizontal components acting separately. Since the cross term has the same temperature dependence as the second term in (A8), we regard it as part of the vertical scattering component.

We introduce now the explicit form of the spectral density for localized spin fluctuations[1]

$$\bar{A}_n(\omega) = a_n \frac{\tilde{\omega}}{1 + \tilde{\omega}^2} \ , \tag{A9}$$

where a_n is a constant independent of energy

$$a_n = 2c\alpha\delta u \left({}_n\bar{\chi}_{2R} + \frac{{}_n\bar{\chi}_{2I}}{\alpha\delta u\,\bar{\chi}_I} \right) \tag{A10}$$

and

$$\tilde{\omega} = \omega/k_B T_S = \alpha\delta u\,\bar{\chi}_I \ . \tag{A11}$$

δU is the increase in the intra-atomic exchange interaction in the impurity cell, and α is the susceptibility enhancement factor for localized spin fluctuations:

$$\alpha = (1 - \delta U \bar{\chi}_R)^{-1} \tag{A12}$$

and

$${}_n\bar{\chi}_{2R} = (1/k_F^{n+1}) \int_0^{2k_F} dq\, q^n |F(q)|^2 \chi_R^2 \ , \tag{A13}$$

$${}_n\bar{\chi}_{2I} = (1/k_F^{n+1}) \int_0^{2k_F} dq\, q^n |F(q)|^2 2\chi_R\chi_I \ , \tag{A14}$$

$$\bar{\chi}_R = (1/N) \textstyle\sum_{\vec{\mathrm{q}}} \chi_R \ , \tag{A15}$$

$$\bar{\chi}_I = (1/N)\sum_{\vec{q}} \chi_I \ , \tag{A16}$$

where χ_R and χ_I are the real and imaginary parts of the generalized susceptibility $\chi(q, \omega)$ in the host metal, and N is the total number of atomic cells.

Defining dimensionless variables

$$\tilde{W} = W_m L_0 / \rho_0 k_B a_3 \ , \tag{A17}$$

$$\tilde{T} = T/T_S \ , \tag{A18}$$

and substituting in (A8), we get

$$\tilde{W} = \tilde{W}_2 + \frac{3}{\pi^2}\left(\frac{a_1}{a_3} - \frac{1}{6}\right)\tilde{W}_4 \ , \tag{A19}$$

where

$$\tilde{W}_n = \frac{1}{\tilde{T}^n}\int_0^\infty d\tilde{\omega}\, \frac{e^{\tilde{\omega}/\tilde{T}}\,\tilde{\omega}^n}{(e^{\tilde{\omega}/\tilde{T}} - 1)^2(1 + \tilde{\omega}^2)} \ . \tag{A20}$$

Taking the limit $\tilde{T} \to 0$, we obtain a linear dependence on $\tilde{T}$ corresponding to the $\tilde{T}^2$ dependence of the electrical resistivity in this limit,

$$\tilde{W}(\tilde{T} \to 0) = \left[\frac{\pi^2}{3} + \frac{4\pi^2}{5}\left(\frac{a_1}{a_3} - \frac{1}{6}\right)\right]\frac{T}{T_S} \ , \tag{A21}$$

which is equivalent to the result of SSM[3]. In the limit $\tilde{T} \to \infty$,

$$\tilde{W}(\tilde{T} \to \infty) = \frac{\pi}{2} + \frac{T_S^2}{T^2}\,\frac{3}{\pi^2}\left(\frac{a_1}{a_3} - \frac{1}{6}\right)\int_0^{\tilde{\omega}_m} \frac{\tilde{\omega}^2\, d\tilde{\omega}}{(1 + \tilde{\omega}^2)} \ . \tag{A22}$$

Hence, at high temperatures the horizontal component gives a constant resistivity, while the vertical component goes to zero as T^{-2}. Note that the general expression (A9) for the spectral density shape becomes inaccurate for large energies, especially for $n = 1$. We introduce a cutoff to prevent an unphysical infinity in the integral of (A22), in which the ω^2 weighting factor for vertical processes enhances the contribution of higher energies. The magnitude of the vertical-process term above T_S as it decreases towards zero depends on the shape of the high-energy tail of the excitation spectrum and cannot be represented by a universal curve.

At high temperatures the local enhancement factor α decreases, leading to a decrease of electrical resistivity below the linear law.[1] Since a_3 defined by (A10) is roughly proportional to α, from (A17), W_m will reflect directly the decrease in α, i.e., the normalizing factor for $\tilde{W}$ will decrease at high temperatures. Thus, the temperature dependence of W_m is rather complex. After a linear increase with T at low temperatures, W_m becomes more or less constant above $\frac{1}{2}T_s$; then, at still highter temperatures it decreases as α decreases.

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